Reevaluating the Friedmann Model: New Redshift Relations for Galaxy Formation and Cosmic Evolution

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Abstract

The Friedmann-Lemaître-Robertson-Walker (FLRW) model, a cornerstone of the Lambda Cold Dark Matter (CDM) framework, has long provided a robust explanation for the formation and evolution of cosmic structures, aligning with much of the observational data. At its core lies the cosmological principle, which posits that the universe is homogeneous and isotropic on large scales. However, recent three-dimensional catalogs present a different picture, revealing a universe that appears non-homogeneous and non-isotropic even at the farthest observational distances. This challenges the FLRW model's accuracy and suggests a need for its reassessment. ,,,,,,,,,,,,,,,,,,,,,,In this study, we derive new redshift-light intensity and redshift-number density relations using the Einstein Field Equations (EFE), employing the galaxy number count method to explore the universe's dynamics and evolution within the FLRW framework. Our findings show that these new relations effectively describe galaxy formation and evolution, enriching our understanding of the cosmos. Notably, they replicate the early burst of galaxy formation, which is consistent with other models. However, at higher redshifts, our results indicate a slower rate of structure formation and distribution, contrasting with predictions from alternative models. Furthermore, our simulation results align with observational data, supporting the FLRW model. These relations also open promising avenues for future cosmological research.

Introduction

The origin and evolution of the universe is a subject of cosmology. Modern physical cosmology is anchored on the Friedmann model, a model that has successfully described the origin and evolution of structures in the universe in line with observations. The backbone of this model is the cosmological principle which states that the universe is homogeneous and isotropic on large scales [1]. Current redshift surveys probing deeper into the universe reveal structures that are inhomogeneous all through length scales casting doubts on the validity of CP and FLRW model in general. This raises the question as to whether or not our universe is indeed Friedmann on large scale. This question motivates our present study to describe relativistic dynamics and structure formation in the Friedmann Universe based on FLRW model. To answer this question, we closely followed recent works [2, 3] which used the conservation principle pertinent to a matter-dominated Friedmann universe to describe the universe's structure formation and evolution. In Contrast to these works, we shall consider the density and cosmic scale factor as variables rather than constants.

Methodology

Light from an astronomical object located at $r(t_e)$, emitted at t_e and received by an observer positioned at $r(t_o)$ at t_o was considered. As this light transverses space, it is assumed to be redshifted. It was also assumed that light intensity has measured redshift and solid angle (θ) and also counted the number of galaxies (n) observed at a given redshift interval dz. Additionally, it was assumed that during the observation time; no new galaxies appeared or disappeared and that the luminosity of every star/galaxy remained constant. The other assumption taken involved the availability of a substantial and direct dataset encompassing various astronomical objects(galaxies) and free from assumptions about the background geometry or uncertainties associated with distance measurements. The specific astronomical quantities we considered were:

1. . Light intensity (I) from an astronomical object e.g., a star or galaxy.
2. . Redshift (z) of the light intensity from the given astronomical object in (1) above in a given direction.
3. . The number density (n) per solid angle (θ) of a class of objects in a given direction.

This information was used in the FLRW metric to derive relevant Einstein field equations of relativistic dynamics and structure formation in the Friedmann universe. We then employed MATLAB programs to interpret the interrelations and deduced model before terminating with a comparison with galaxies' observational data retrieved from NASA/IPAC Extragalactic Database (NED) for redshifts between 0 and 5

Analytical Results

It has been shown that the Einstein field equations within the FLRW framework [2] are given by;

$$12kc^2 + 3R'(t)^2 = \beta c^4 \rho(t)R(t)^2 - \lambda c^2 R(t)^2 \tag{1}$$

$$4kc^2 + 2R(t)R''(t)^2 + R'(t)^2 = \beta c^2 p(t)R(t)^2 - \lambda c^2 R(t)^2 \tag{2}$$

Rearranging and simplifying equation (1) gives

$$dt = \frac{dR(t)}{\sqrt{\frac{(\beta c^4 \rho c^2)R(t)^2}{3} - 4kc^2}} \tag{3}$$

This equation differentiated our model from other competing models adopted in this work and was used to describe relativistic dynamics and structure formation in the Universe. The equation was applied in deriving the light intensity-redshift and number density-redshift relations.

Light Intensity-Redshift Relation

Light from an astronomical object commencing at $r(t_e)$ and progressing towards the spatial origin $r = r(t_o)$ at time $t = t_o$ was considered. Straightforward calculations based on equation (3) and the FLRW metric for a null geodesic:

$$0 = c^2dt^2 - \frac{R(t)^2}{(1 + kr(t)^2)^2}dr^2 \tag{4}$$

yields:

$$r(z) = \frac{2((1+z)(aR(t_o) + \sqrt{a^2R(t_o)^2 - 4k}) - (aR(t_o) + \sqrt{a^2R(t_o)^2 - 4k(1+z)^2}))}{(aR(t_o) + \sqrt{a^2R(t_o)^2 - 4k})(aR(t_o) + \sqrt{a^2R(t_o)^2 - 4k(1+z)^2}) + 4k(1+z)} \tag{5}$$

where $k = -1, 0, 1.$

Following the procedure of earlier works [2, 3] light intensity redshift relation is easily shown to be;

$$I(z) = \frac{L(1 + kr(z)^2)^2}{4(1+z)r(z)^2\pi R(t_o)^2} \tag{6}$$

Substituting equation (5) into equation (6) generates our desired light intensity-redshift relation.

Number Density-Redshift Relation

Let N symbolize the number of galaxies existing per unit volume within a spatial metric denoted as $((dr^2 + r^2d\theta^2 + r^2sin^2\theta d\theta))(1 + kr^2)^2$ and the volume element be given by $(r^2sin\theta d\theta d\phi dr)(1 + kr^2)^3$. Following earlier procedures, It can easily be shown that the number of galaxies between coordinate hyperspheres $r(z)$ and $r(z + dz)$ [2] is given by

$$n(z)dz = \frac{4\pi r(z)^2 Nr'(z)dz}{(1 + kr(z)^2)^3} \tag{7}$$

where $r(z)$ is given by Equation (5) and $r'(z)$ is the derivative of equation (5) concerning z. Equation (7) is our desired number density-redshift relation for describing the universe's relativistic dynamics and structure formation. Equations (6) and (7) constitute the most fundamental results in this work. The next section will use these equations to model our universe by making MATLAB plots.

Graphical Results

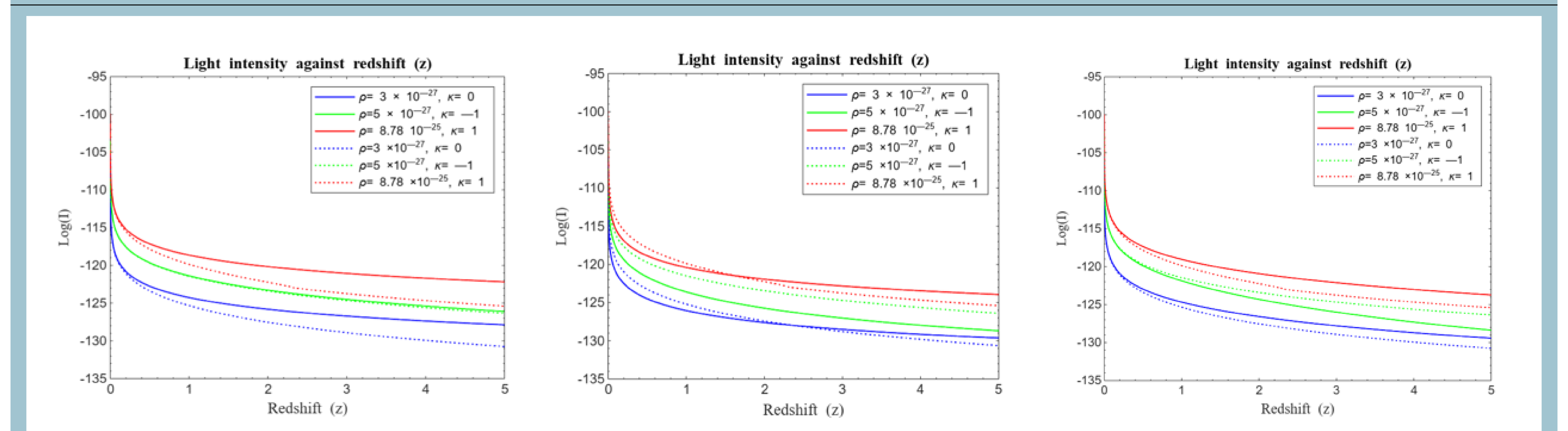

Figure 1. Our model's light intensity-redshift relation comparison with: Left panel-approximate redshift model [2], Middle panel-parametric model, Right panel-nonparametric model [3]. The dotted curves represent our model

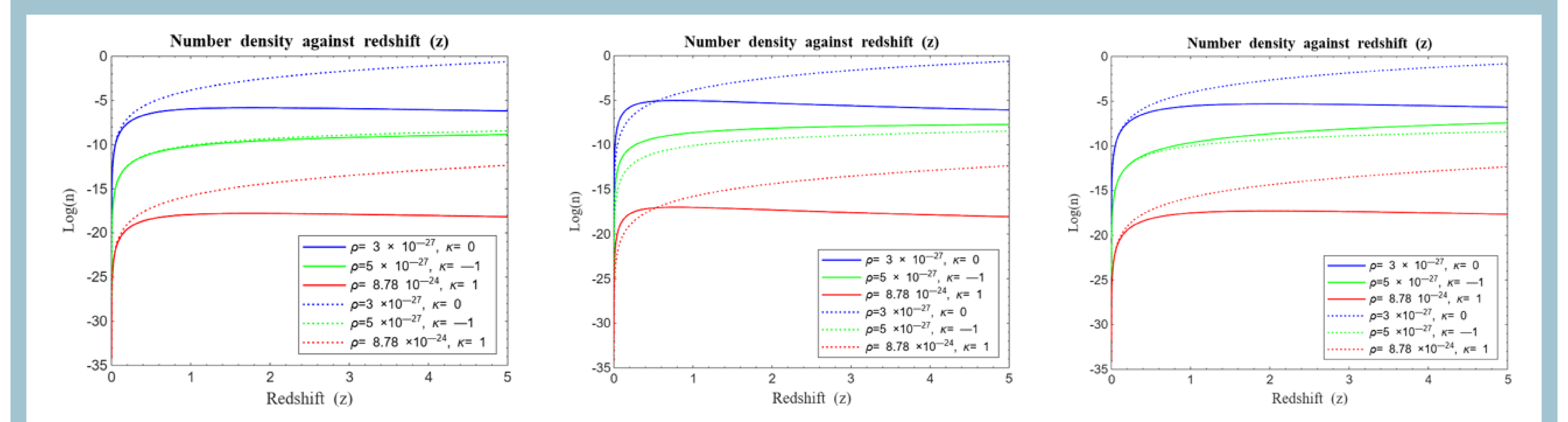

Figure 2. Our model's number density-redshift relation comparison with: Left panel-approximate redshift model [2], Middle panel-parametric model, Right panel-nonparametric model [3]. The dotted curves represent our model

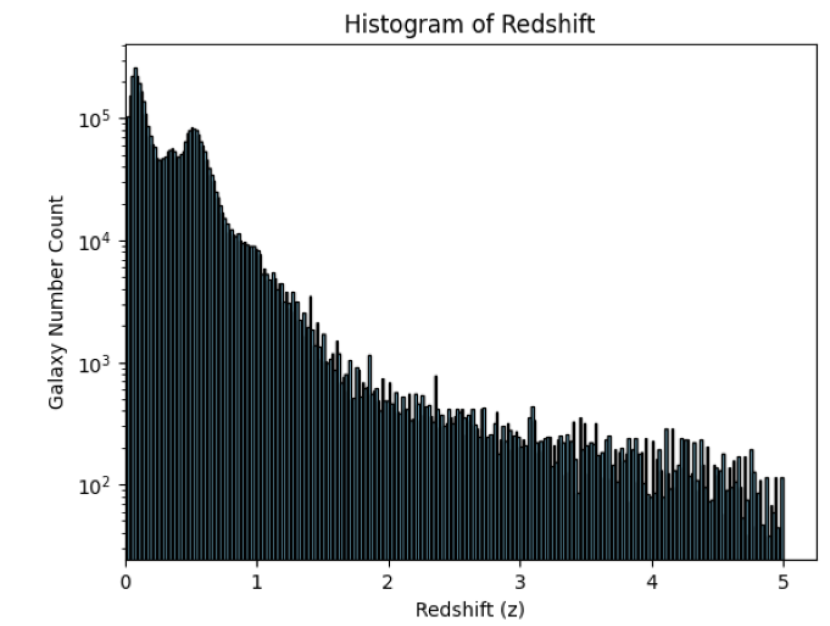

Figure 3. Redshift distribution for NED observed galaxies in the infrared spectral region

Discussion

Figure 1 compares our model with three different models from earlier studies[2][3]. The figure shows that light intensity curves decayed exponentially with increasing redshift due to a consequence of photon dilution. As the universe expands, the wavelength of photons also stretches leading to a decrease in the energy density of the photons. Light intensity decays slower at lower densities compared to higher ones. A rapid decrease in intensity is observed at redshifts $z \leq 0.7$ then slowly decays as the redshifts increase in all the curvatures. At lower redshifts, the universe is relatively closer in the cosmic timeline in contrast to other models where the intensity attenuated faster at redshifts $z \leq .1.0$ before slowly decaying. Our model showed a faster attenuation of light as redshifts increased.

Figure 2 shows a higher activity in galaxy formation rate at the beginning of the universe than at a much later time. Formation of cosmic structures seems to have been faster at redshifts $z \leq 0.7$ after which it slowed down. Our model showed the formation of structures continued as redshift increased contrary to previous studies. Various factors, including the density fluctuations in the early universe, the nature of dark matter, and the role of dark energy influence galaxy formation.

The observational results in Figure 3 show that the number of galaxies formed faster at the beginning of the universe at redshifts $0 \leq z \leq 0.7$ and gradually slowed down. The general trend of the formation of galaxies as seen from this figure is consistent with our theoretical model.

Summary and conclusion

For a long time, scientists have successfully explained structure formation in the universe using the standard model. However, current 3D maps of the universe raise doubts about this model as galaxy redshift surveys reveal more inhomogeneous structures on large scales. These maps depend on cosmic distance measurements, which are prone to errors. In this work, we derived Einstein field equations for a Friedmann universe considering dark energy effects. Our analytical and computational results show that light intensity decreases with redshift, aligning with classical physics. Our simulation results are in agreement with observational data, indicating the Friedmann model's validity. Furthermore, our relations stand as promising candidates for cosmological probes in the future.

References

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