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I would like to express my sincere gratitude to AFAS for the grant that made the presentation of this research possible.

ABSTRACT

The recent measurements of Baryon Acoustic Oscillations from the Dark Energy Spectroscopic Instrument survey (DESI DR2 BAO) and the distances of Type Ia supernovae (SNIa) have consistently suggested that dark energy evolves with time. This work investigates time-evolving dark energy in Rastall gravity at the background cosmological level. We constrain Rastall gravity, the wRastall gravity Λ CDM model, and the wCDM model, and perform further statistical analyses using recent measurements: DESI DR2 BAO, supernova compilations (PantheonPlus, Union3, and DESY5), and cosmic chronometer data (31 points from the relative ages of massive, early-time, passively evolving galaxies plus 3 new points from DESI 2024). From the Rastall gravity framework, the matter component is injected into dark energy at all times for cosmic acceleration, and dark energy varies through cosmic time even when $w_{de} = -1$. The results also show that late-time dark energy in Rastall gravity is quintessence-like: the effective equation-of-state parameter in the wRastall model remains $w_{de} > -1$ even into the future (up to $z \approx -1$). The model also indicates a longer matter-dominated era and a delayed onset of cosmic acceleration in Rastall gravity compared to the Λ CDM model. After the comprehensive statistical analysis, we observe that the model is favored by late-time cosmological measurements, suggesting that it is a viable description of the universe's late-time acceleration.

INTRODUCTION

Theoretical overview and Methods

- The Rastall gravity is one of the alternative theory of GR, and it was proposed in 1972 by Rastall, P. (1972), which has a non-conserved stress-energy tensor that serves as the source of the gravitational field.
- In the limiting case, Rastall gravity is equivalent to GR (Abbas & Shahzad, 2020). Taking the divergence of Rastall's equation with a new energy-momentum tensor and it's experiencing a notable increase in popularity in exploring different aspects of cosmology.
- Recent findings from the DESI Data Release 2 (DR2) reveal evidence for dynamical dark energy, challenging the Λ CDM model (Adame et al., 2025).
- In this paper, the comprehensive analysis of the characterization of dynamical dark energy in Rastall gravity has been constrained together with the Λ CDM and wCDM models using the recent measurements.

- Taking the divergence of Rastall's equation with a new energy-momentum tensor (Darabi et al., 2018)

$$G_{\mu\nu} + \kappa \lambda g_{\mu\nu} R = \kappa T_{\mu\nu}$$

where κ is the gravitational coupling constant modified according to Rastall theory. In the limit $\kappa \lambda g_{\mu\nu} R \rightarrow 0$, the standard theory of General Relativity is recovered.

- The conservation equations for Rastall gravity (Darabi et al., 2018)

$$T^{\mu\nu}_{;\mu} = 0 \Rightarrow T^{\mu\nu}_{;\mu} = \frac{(\lambda-1)}{16\pi G} R^{;\nu}$$

For $\lambda = 1$, the Rastall gravity is equivalent to GR.

- In straightforward expression, the energy densities of dark matter and dark energy become :

$$\dot{\rho}_m + 3H\rho_m = 0, \\ \dot{\rho}_{de} + 3H(1+w_{de})\rho_{de} - \frac{\gamma-1}{2}[(1-3w_{de})\dot{\rho}_{de} + \dot{\rho}_m] = 0.$$

- The modified Friedmann equation: where the parameter

$$3H^2 = \alpha \rho_{m0} a^{-3} + \rho_{de0} a^\beta$$

$$\alpha = \left[1 + \frac{(1-\gamma)}{2w_{de} + (\gamma-1)(1-3w_{de})} \right], \text{ and } \beta = \frac{6(1+w_{de})}{2 - (\gamma-1)(1-3w_{de})}$$

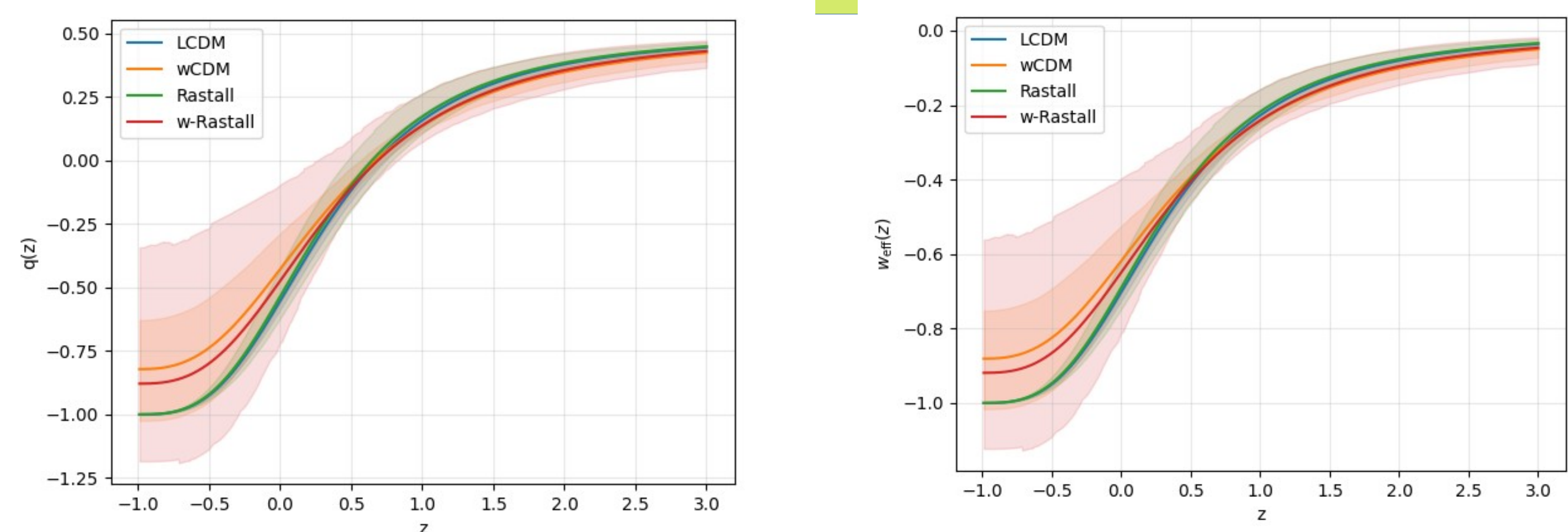
DATA

- DESI DR2 BAO, supernova compilations (PantheonPlus, Union3, and DESY5), and cosmic chronometer data (31 points from the relative ages of massive, early-time, passively evolving galaxies plus 3 new points from DESI 2024).

- We first focus on constraining the cosmological parameters using late-time cosmological measurements and the Python libraries EMCEE (Qi et al., 2023) and GetDist (Lewis, 2019).

- To constrain the cosmological parameters we apply Monte Carlo Markov Chain (MCMC) simulation.

RESULTS



From Fig.1 we clearly observe the late-time dominance of dark energy in wRastall gravity, as shown by the effective equation-of-state parameters and the deceleration parameter.

From Fig.2, the best fit values of w for both models are greater than negative one, which indicates the wRastall gravity and wCDM represent the quintessence phase of the Universe, and suggests that the dark energy is varying with cosmic time. This also well represented by the MCMC contour plots.

From these plots, we observe that the equation-of-state parameter remaining greater than negative one in two models also indicates that the future universe is staying in the quintessence regime.

Conclusion

Our findings demonstrate that, within modest sigma deviations, the inferred Hubble parameter values from all models under consideration are statistically consistent with both Planck 2018 and SH0ES data. Rastall-type cosmologies, along with Λ CDM and wCDM, may help reduce the Hubble tension at the level probed by late-time observations, as the model predictions are significantly closer to both direct and indirect H0 estimates than the well-known difference between Planck and SH0ES. The current research shows that modified gravity scenarios, such as Rastall gravity, are nonetheless observationally feasible and competitive with traditional cosmology, even though a definitive resolution requires accounting for all CMB limitations. The consistent tendency for values greater than -1 in the limits on the dark energy equation-of-state parameter for both wCDM and wRastall models indicates

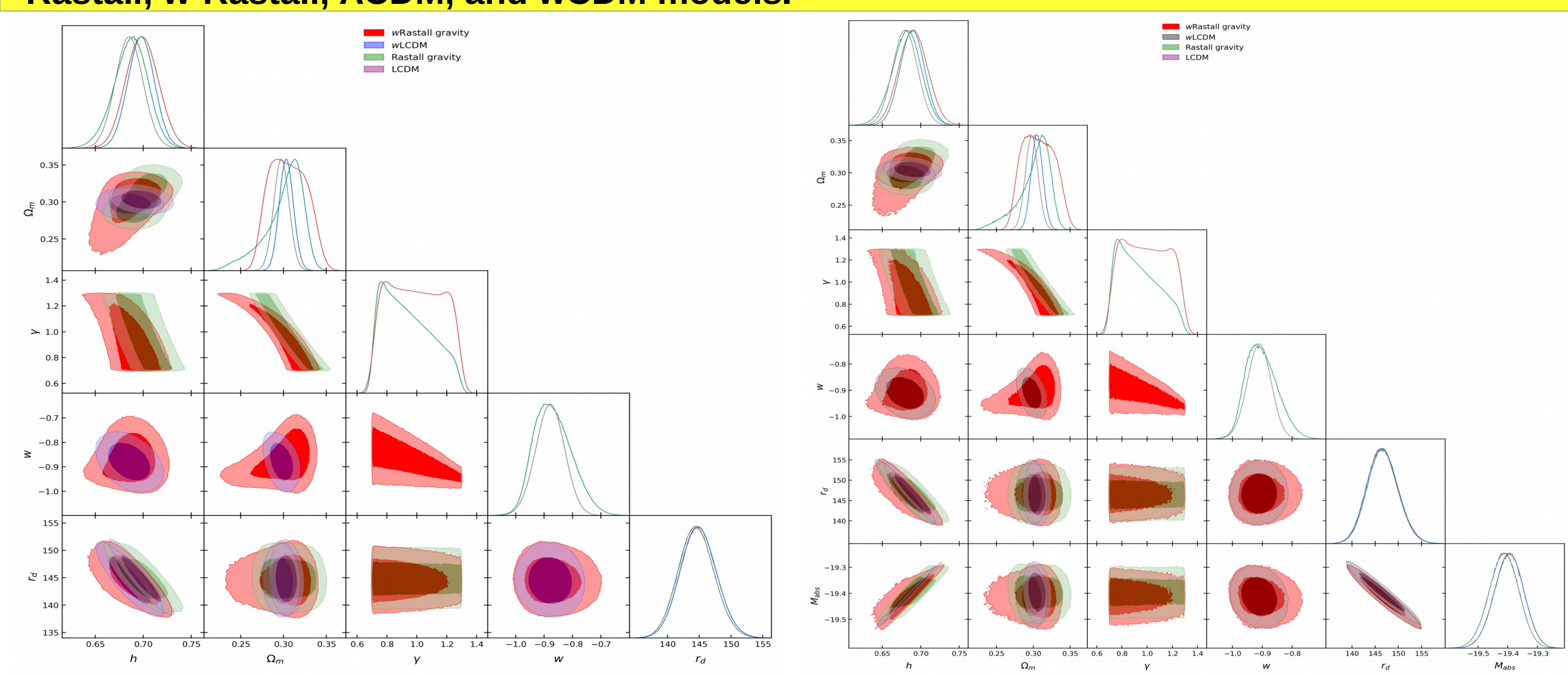


Fig 2: The constraining parameters for Rastall, w-Rastall, Λ CDM, and wCDM models at 68% and 95% level of Confidence space for the posterior parameters for DESI DR2 BAO +CC measurements.