

# Multi-Wavelength Modelling of Low-Luminosity Gamma-Ray Burst Afterglows in the Forward-Shock Synchrotron Framework



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## Abstract

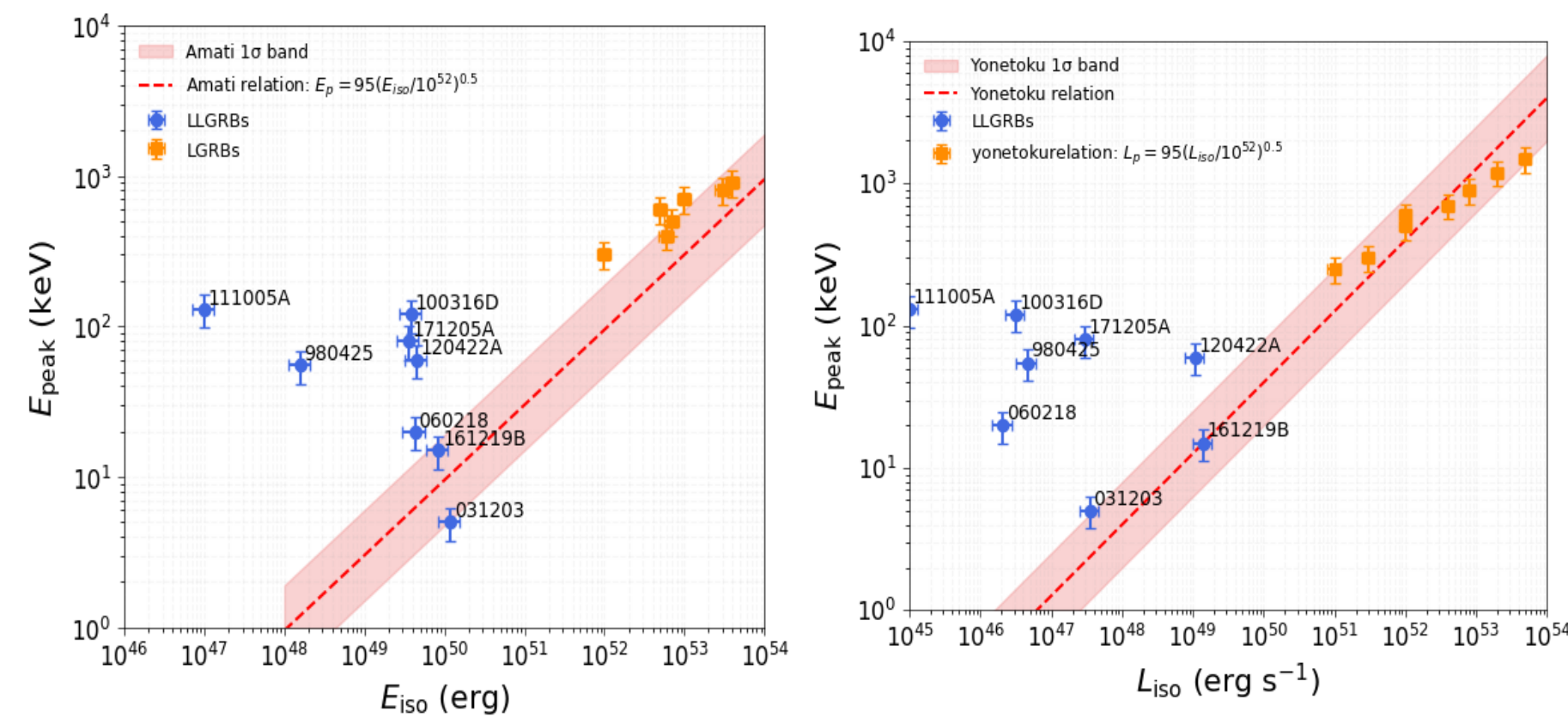
We present a multi-wavelength modeling analysis of Low-Luminosity Gamma-Ray Burst (LLGRB) afterglows within the standard forward-shock synchrotron framework. Synthetic light curves are generated across radio ( $10^9$  Hz), UV ( $10^{15}$  Hz), X-ray ( $10^{18}$  Hz), and  $\gamma$ -ray ( $10^{20}$  Hz) bands using physically motivated parameters. The model predicts a common peak time of  $\sim 0.01$  days across radio, UV, and X-ray bands. Radio, UV, and X-ray bands follow similar decay slopes ( $\alpha \approx -1.36$  to  $-1.38$ ), indicating a shared slow-cooling synchrotron regime, while  $\gamma$ -rays decline more steeply ( $\alpha \approx -6.2$ ). When compared to observed LLGRB data, our modeling reproduces the strong  $E_{\text{iso}}-L_{\text{iso}}$  correlation and the  $E_{\text{iso}}-z$  dependence reported in the literature. These results support the forward-shock model as a robust explanation for broadband GRB afterglows.

## Introduction

### What Are LLGRBs?

- $E_{\text{iso}} = 10^{48} - 10^{50}$  erg (2–4 orders below classical GRBs)
- $z = 0.3$  (predominantly local Universe)
- $T_{90} > 10$  s, smooth single-peaked light curves
- **Soft spectra**,  $E_{\text{peak}}$  often in X-ray band
- **Supernova association**: Type Ic-BL (GRB 980425/SN 1998bw, GRB 060218/SN 2006aj)
- **Mildly relativistic**:  $\Gamma = 2-10$  (vs.  $\Gamma = 100$  for classical GRBs)

**Scientific Motivation:** LLGRBs provide unique insights into relativistic outflows, shock microphysics, and the GRB-supernova connection. Their high local rate makes them potential sources of ultra-high-energy cosmic rays.



**Figure 1:** (Left) Amati relation ( $E_{\text{peak}}$  vs.  $E_{\text{iso}}$ ) and (Right) Yonetoku relation ( $E_{\text{peak}}$  vs.  $L_{\text{iso}}$ ). LLGRBs occupy the low-energy extension of both correlations.

**AIM:** To model multi-wavelength afterglows for LLGRBs, by exploring empirical correlations among isotropic energy ( $E_{\text{iso}}$ ), luminosity ( $L_{\text{iso}}$ ), flux ( $F_{\text{peak}}$ ), and evaluate how the observed light curves compare to the theoretical forward-shock synchrotron framework.

## Theoretical Framework

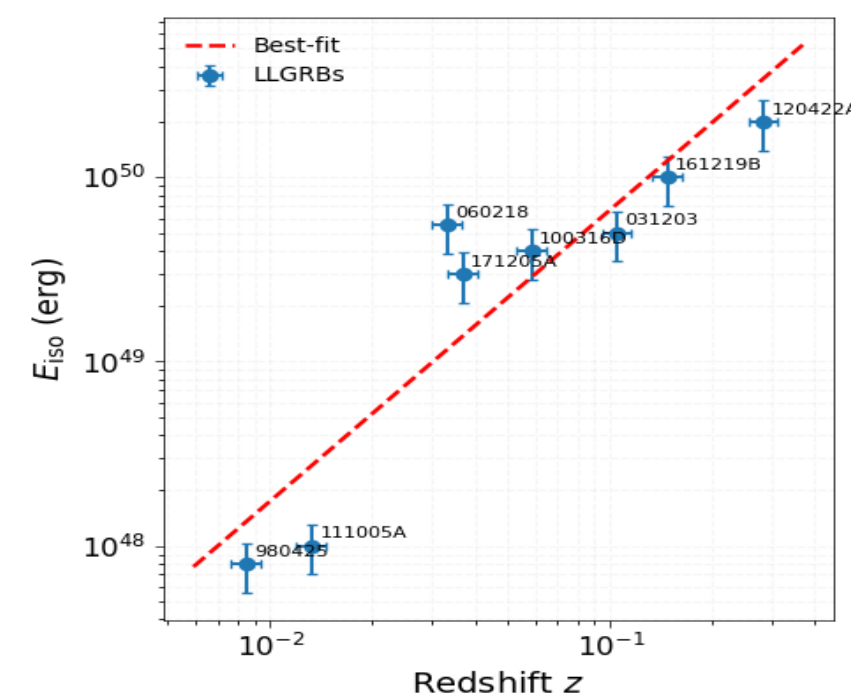
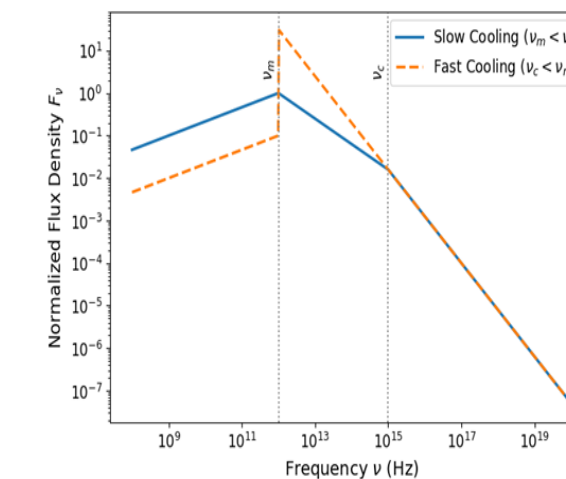
**Forward Shock Dynamics:** For a blast wave expanding into a uniform interstellar medium (ISM), the Lorentz factor  $\Gamma$  evolves with observer time  $t$

$$\Gamma(t) \approx 6.2 \left( \frac{E_{\text{iso},49}}{n_0} \right)^{1/8} \left( \frac{t}{10^3 \text{ s}} \right)^{-3/8} (1+z)^{3/8}$$

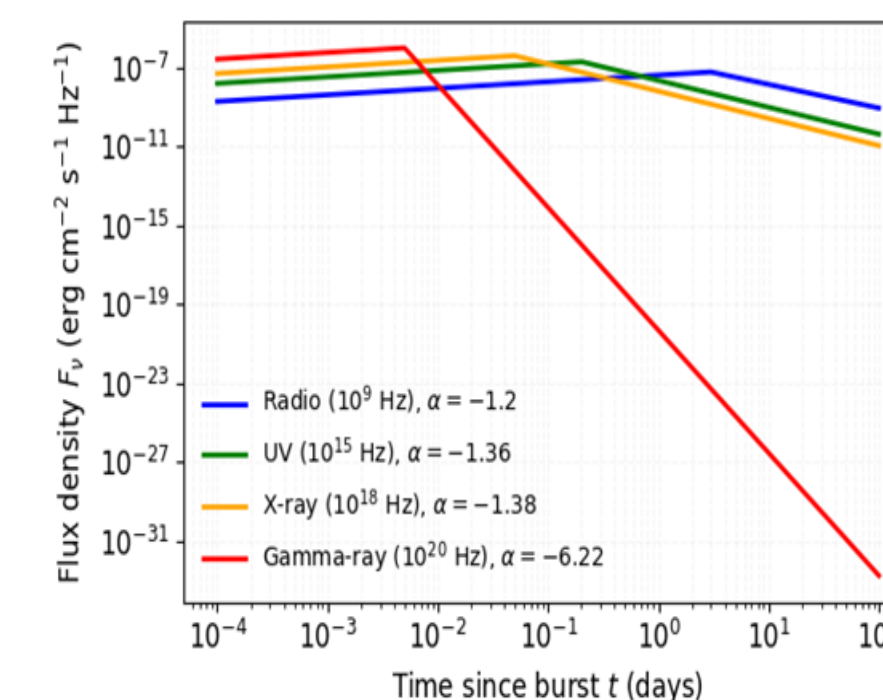
**Characteristic Synchrotron Frequencies:** The general form for the synchrotron flux density at a given time is given by:

$$F_\nu(t) = F_{\nu,\text{max}}(t) \times S(\nu, \nu_m(t), \nu_c(t))$$

$$S(\nu, \nu_m, \nu_c) \propto \begin{cases} \left(\frac{\nu}{\nu_m}\right)^{1/3}, & \nu < \nu_m, \\ \left(\frac{\nu}{\nu_m}\right)^{-(p-1)/2}, & \nu_m < \nu < \nu_c \\ \left(\frac{\nu_c}{\nu_m}\right)^{-(p-1)/2} \left(\frac{\nu}{\nu_c}\right)^{-p/2}, & \nu > \nu_c. \end{cases}$$



**Figure 2:** Left: Isotropic-equivalent energy  $E_{\text{iso}}$  as a function of redshift  $z$ . The correlation suggests a luminosity-dependent detection bias or intrinsic evolution. Right: Broadband spectrum at  $t = 1$  day. Spectral indices are consistent with  $p \approx 2.2$  in the slow-cooling regime.



## Data and Methods

### Model Parameters

- $E_{\text{iso}} = 10^{48} - 10^{50}$  erg
- $n_0 = 0.1 - 10 \text{ cm}^{-3}$
- $p = 2.2$
- $\epsilon_e = 0.1, \epsilon_B = 0.01$
- $z = 0.0085 - 0.283$  (from observed LLGRB sample)

### Observed LLGRB Sample (8 bursts)

GRB 980425, GRB 031203, GRB 060218, GRB 100316D, GRB 171205A, GRB 111005A, GRB 120422A, GRB 161219B

## Results

### Temporal Decay

- Common peak time:**  $t_p \approx 0.01$  days (radio, UV, X-ray)
- Decay slopes:**  $\alpha \approx -1.36$  to  $-1.38$  (radio, UV, X-ray)
- $\gamma$ -ray decay:**  $\alpha \approx -6.2$  (prompt emission cessation)
- Matches theory:  $\alpha = (3p-3)/4 \approx -1.4$  for  $p = 2.2$

### Spectral Indices

- $\beta \approx -0.6$  ( $\nu_m < \nu < \nu_c$ )
- $\beta \approx -1.1$  ( $\nu > \nu_c$ )

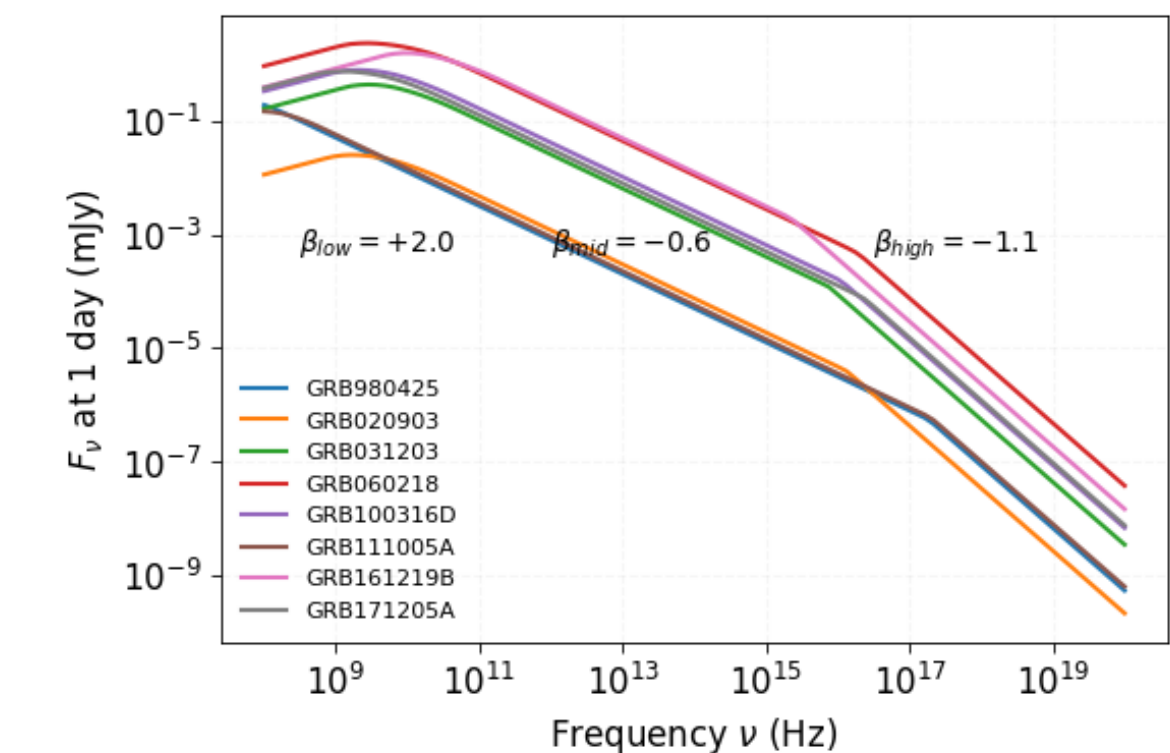
## Discussion

### Physical Interpretation

- Slow-Cooling Regime:**  $\alpha \approx -1.37$  indicates  $\nu_m < \nu_{\text{obs}} < \nu_c$
- Electron Index:**  $p \approx 2.2$  (diffusive shock acceleration)
- Uniform ISM:** Favored over wind-like profile

### LLGRBs in Context

- Distinct Population:** Weaker jets or larger radii
- Continuum Population:** Low-luminosity tail of classical GRBs (off-axis viewing)
- Strong  $E_{\text{iso}}-L_{\text{iso}}$  correlation suggests stable radiative efficiency.



**Figure 2:** Broadband afterglow spectrum of a representative LLGRB constructed at a fixed epoch ( $t=1$  dy after the burst). The flux density  $F_\nu$  is shown as a function of observing frequency ( $\nu$ ) on log scale regimes

## SUMMARY & FUTURE DIRECTIONS

- Forward-shock model reproduces LLGRB afterglows
- Radio, UV, X-ray: common peak ( $\sim 0.01$  days), similar decay ( $\alpha \approx -1.37$ )
- $\gamma$ -ray decays steeply ( $\alpha \approx -6.2$ )
- Spectral indices imply  $p \approx 2.2$ , slow-cooling regime
- LLGRBs occupy low-energy extension of Amati/Yonetoku relations
- Strong  $E_{\text{iso}}-L_{\text{iso}}$  correlation

### Future Work

- Bayesian parameter inference with multi-wavelength data
- Systematic LLGRB vs. classical GRB comparison
- Expand sample with SVOM, Einstein Probe
- Include inverse Compton/SSC components

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