Quantum Simulation of Astrophysical Phenomena: Analogue Blackholes and Hawking Radiation in Exciton-Polariton Condensates

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Background and Purpose

In a seminal work, Unruh proposed acoustic black holes as a platform for probing quantum gravitational phenomena, such as Hawking radiation, within laboratory-based analogue systems.

The Model

Exciton-polariton condensate dynamics are governed by a dissipative Gross–Pitaevskii equation coupled with a reservoir rate equation:

$$i\frac{\partial\psi_l}{\partial t} = \left[-\frac{1}{2}\nabla^2_{2D} + |\psi_l|^2 + h|\psi_{3-l}|^2 + gn_l + \tilde{g}n_{3-l} + i\left(n_l - \frac{1}{2}\right)\right]\psi_l + (-1)^{3-l}B\psi_{3-l} + \sigma\left(\frac{\partial}{\partial x} + i(-1)^l\frac{\partial}{\partial y}\right)^2\psi_{3-l}$$

$$\frac{\partial n_l}{\partial t} = I_l - \left(\Gamma + G\,|\psi_l|^2\right)n_l, \qquad l = 1,2 \quad (2) \qquad (1)$$

The dynamics of the system are described by the spinor condensate wavefunction $\psi(x,y,t) = [\psi_1, \psi_2]^T$ [cite: 10] and the reservoir evolution $n_l(x,y,t)$[cite: 18].

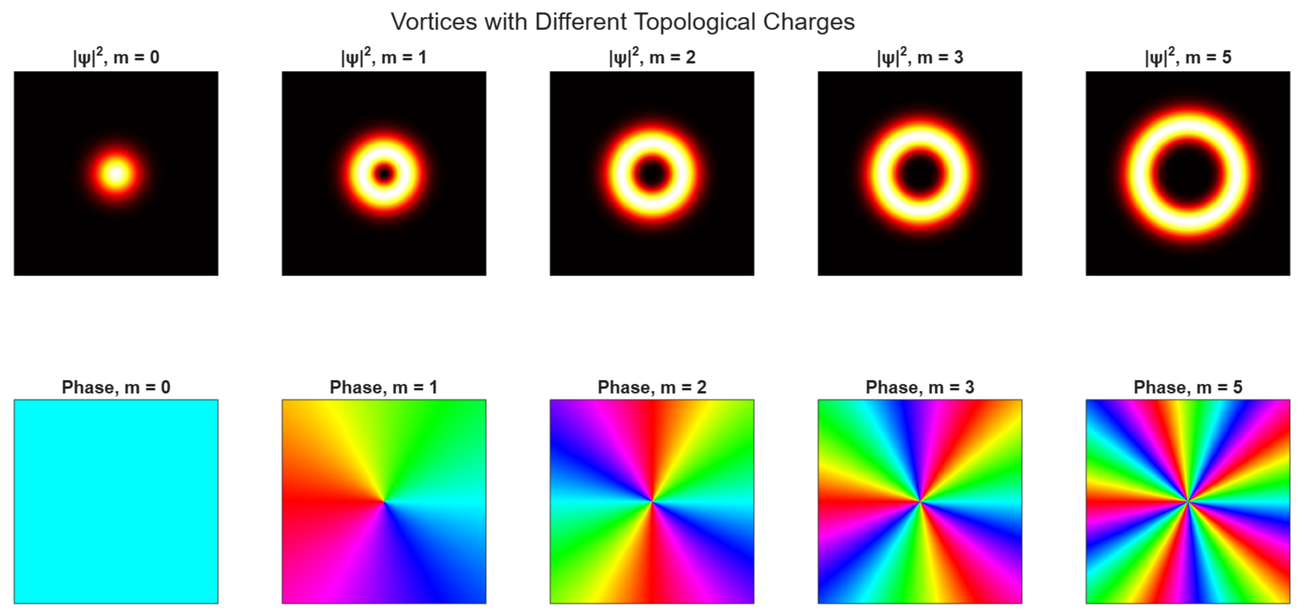
The condensate evolution incorporates **kinetic energy** from particle diffraction ($-\frac{1}{2}\nabla^2_{2D}$) alongside **nonlinear** self- and cross-component interactions ($|\psi_l|^2, h|\psi_{3-l}|^2$).

Magnetic field effects are introduced via the B term, while the σ parameter accounts for spin-orbit coupling or TE–TM splitting. **Non-equilibrium gain and loss** are governed by the interaction between the condensate gain G and the reservoir decay Γ. This reservoir is replenished by **external laser pumping** I_l and depleted through both natural decay and **stimulated relaxation** ($G|\psi_l|^2 n_l$) into the condensate.

Variational approach

To characterize ring vortex solitons in polariton condensates, we employ the following Gaussian trial functions

$$\psi_{1,2}(r,\theta,t) = A_{1,2}(t)r^{|m_{1,2}|}\exp\left[-\frac{r^2}{W^2_{1,2}(t)} + i(m_{1,2}\theta + C_{1,2}(t)r^2 - \mu_{1,2}(t))\right] \qquad (2)$$



using the conservative Lagrangian and dissipative terms

$$L_c = \int_0^{2\pi}\int_0^{\infty}\mathcal{L}_c r\,dr\,d\theta, \quad \frac{d}{dt}\left(\frac{\partial L_c}{\partial \dot{q}}\right) - \frac{\partial L_c}{\partial q} = 2\mathrm{Re}\left(\int_0^{2\pi}\int_0^{\infty}\mathcal{Q}\frac{\partial\psi_l^*}{\partial q}r\,dr\,d\theta\right), \qquad (3)$$

where $\mathrm{Re}(\ldots)$ denotes the real part, and $q = \{A_l(t), W_l(t), C_l(t)\}$, where $l = 1,2$.

Hydrodynamic Variables

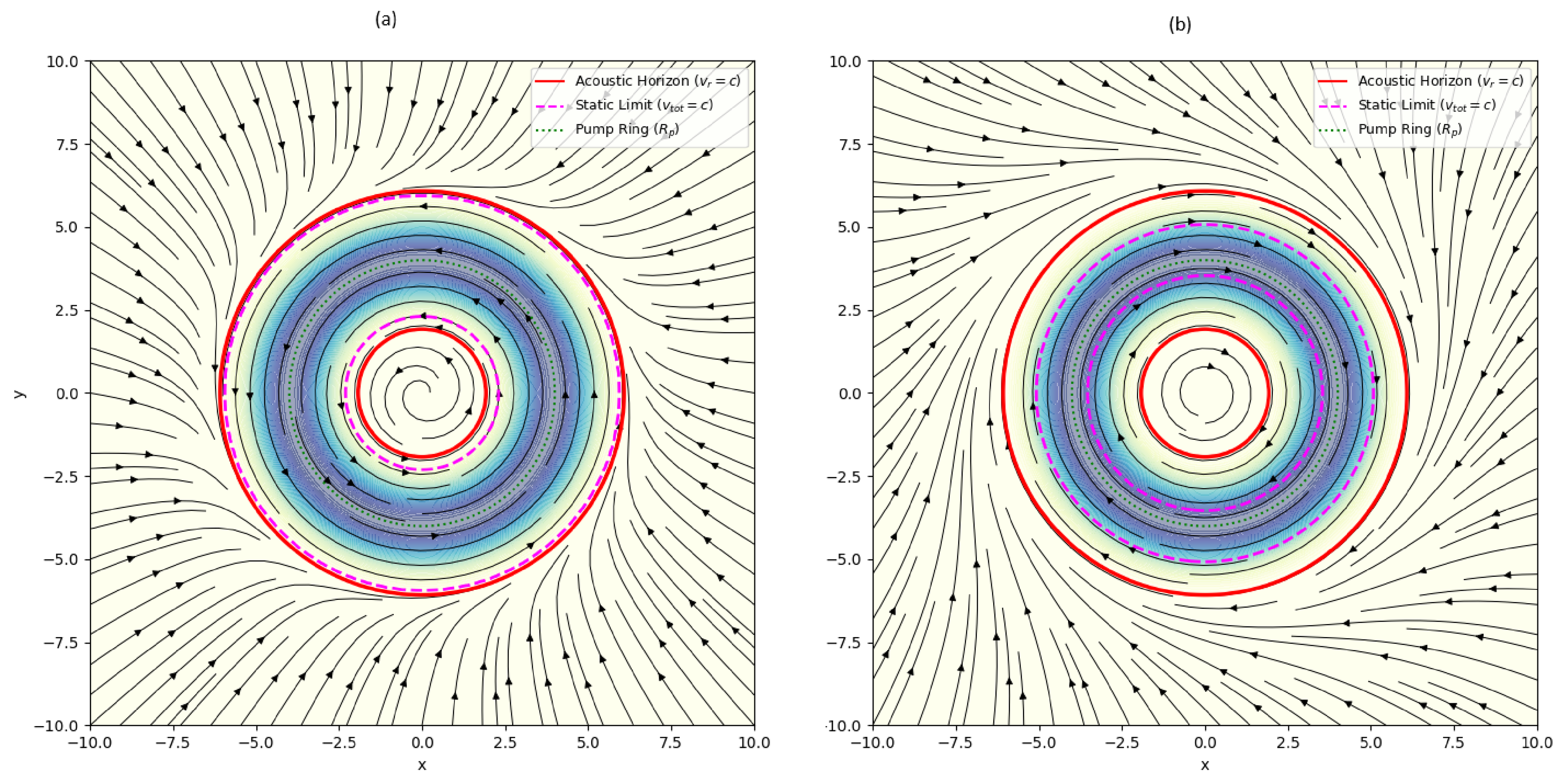
Using the Madelung representation

$$\psi_l = \sqrt{\rho_l(x,y,t)}\exp(i\phi_l(x,y,t)) \quad l = 1,2. \qquad (4)$$

The scaled local speed of sound of the form

$$c(r) = \sqrt{\rho_{1,2}\left(1 + \frac{gGP(r)}{(\Gamma + G\rho_{1,2}(r))^2}\right)}, \qquad (5)$$

Illustrations of acoustic black holes with $m = 2$ and $m = -10$.



Bogoliubov excitations Problem

Analysis is restricted to the neighborhood of the acoustic horizon r_H. We linearize azimuthal fluctuations around steady-state vortex ring solitons:

$$\psi_l(r_H,\theta,t) = A_l\exp(i(m_l\theta - \mu_l t)) \qquad (6)$$

where A_l are amplitudes and m_l are topological charges.
Hawking radiation manifests as collective Bogoliubov modes propagating in opposite directions. We employ the perturbation ansatz:

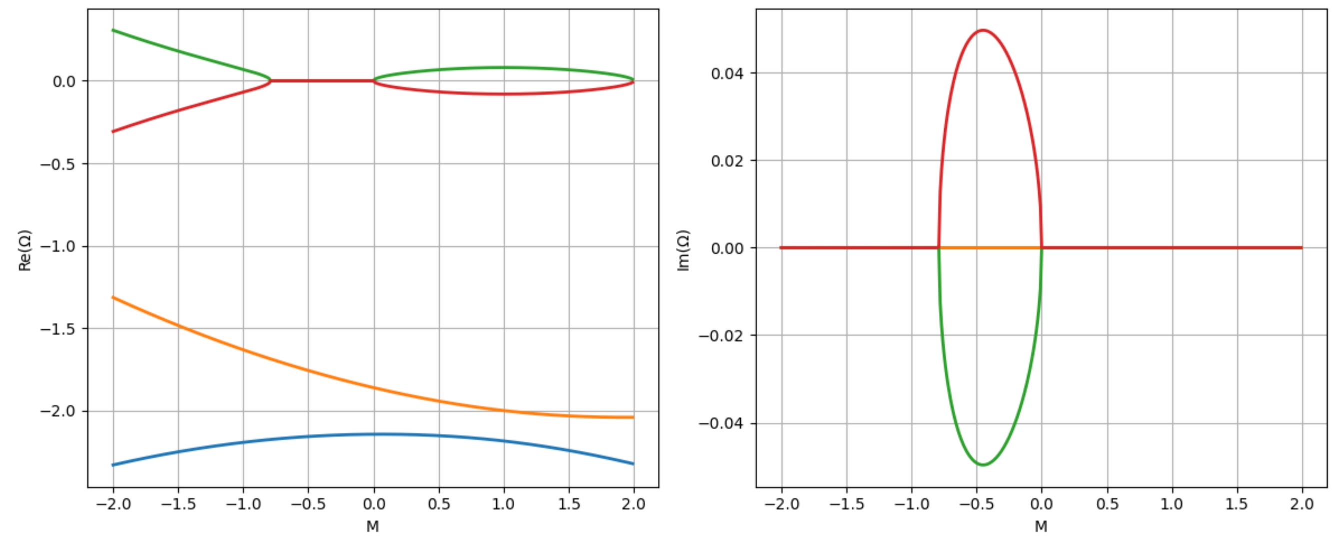
$$\delta\psi_l(\theta,t) = \sum_M [u_l e^{i(M\theta - \Omega t)} + v_l^* e^{-i(M\theta - \Omega^* t)}] \qquad (7)$$

where M is the azimuthal integer and Ω is the complex eigenfrequency. Dispersion Relation The linearized dynamics yield an eigenvalue problem $\mathbf{M}\mathbf{x} = \Omega\mathbf{x}$. The dispersion relation is extracted via:

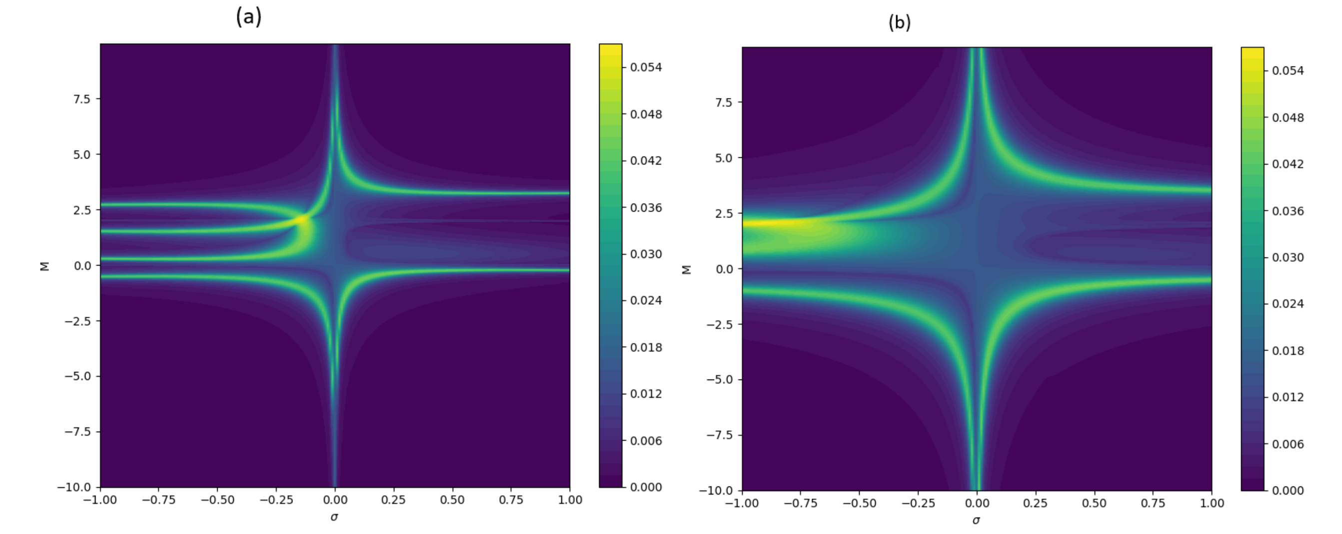
$$\det(\mathbf{M} - \Omega\mathbf{I}) = 0. \qquad (8)$$

Dispersion Relation $(\Omega_{1,2,3,4})$

Distinguishes between escaping Hawking phonons (positive branch) and their trapped, energy-balancing partners (negative branch). Examples using typical system parameters



$\mathrm{Im}(\Omega)$: Governs dynamical stability; positive values represent Modulational Instability, which leads to the fragmentation of the horizon. Exploring stability of the acoustic horizon in the σ, M parameter space



Conclusion

Exciton-polariton condensates can act as "lab-bench universes" where we can recreate and study the physics of black hole horizons in a controlled environment.

References

References

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